

UCL-IPT-98-02

# A Model for a Dynamically Induced Large $CP$ -Violating Phase<sup>12</sup>

D. Delépine

*Institut de Physique Théorique, Université catholique de Louvain,  
B-1348 Louvain-la-Neuve, Belgium*

## Abstract

Assuming a new interaction with a  $\theta$  term for the third generation of quarks, a value of  $\theta$  different from zero dynamically induces the top-bottom mass splitting and a large  $CP$ -violating phase.

In the electroweak standard model, there are 2 independent sources of  $CP$  violation: one is the phase of the quark mixing matrix (the Cabibbo-Kobayashi-Maskawa matrix ( $V_{CKM}$ ))[5] and the second is the strong  $\theta$  angle of QCD[6].

Nowadays, the mechanism for  $CP$  violation is still not yet understood and the  $CP$  puzzles have been turned into questions concerning tiny quark masses and a large  $CP$ -violating phase. The fact that the top quark is very heavy compared to the other quarks,  $m_t = 180 \pm 12$  GeV [1], suggests that the third generation may be playing a special role in the dynamics at the electroweak scale. So we assume the existence of a new interaction playing only with the third generation and strong enough to lead to the formation of quark-antiquark bound states which trigger dynamically the breaking of the electroweak symmetry[2, 3, 4]. This new interaction is conserving isospin symmetry between top and bottom quarks and generates a  $\theta$  term.

We show that in this model, the  $\theta$  term breaks the symmetry between top and bottom as expected from general theorems [11] and induces naturally a

---

<sup>1</sup>This work was done in collaboration with J.-M Gérard, R.Gonzalez Felipe and J.Weyers.

<sup>2</sup>talk presented at the VI Workshop on Particles and Fields (21-27 November 1997 in Morelia Mich., Mexico)

large CP-violating phase in  $V_{CKM}(\delta_{CKM})$  due to the smallness of the  $m_b/m_t$  mass ratio.

## 1 The model

We consider a standard model Higgs sector in combination with an effective new strong interaction acting on the third generation of quarks and characterized by a  $\theta$  term [8, 7].

The total effective Lagrangian of our model is thus given by

$$L = L_H + L_\Sigma + L_\theta , \quad (1)$$

with  $L_H, L_\Sigma$  and  $L_\theta$  defined as follows:

$$L_H = D_\mu H^\dagger D^\mu H - m_H^2 H^\dagger H + \left( h_t \bar{\psi}_L t_R H + h_b \bar{\psi}_L b_R \tilde{H} + \text{h.c.} \right), \quad (2)$$

where  $H = \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$ ,  $\tilde{H} = \begin{pmatrix} H^+ \\ -H^{0*} \end{pmatrix}$  and  $\psi_L = \begin{pmatrix} t_L \\ b_L \end{pmatrix}$ ;  $h_t$  and  $h_b$  are the Yukawa couplings and  $D_\mu$  is the usual covariant derivative of the standard model.  $L_\Sigma$  parametrizes the effects of the new interaction on the top and bottom quark.

$$L_\Sigma = D_\mu \Sigma_t^\dagger D^\mu \Sigma_t + D_\mu \Sigma_b^\dagger D^\mu \Sigma_b - m^2 (\Sigma_t^\dagger \Sigma_t + \Sigma_b^\dagger \Sigma_b) + g (\bar{\psi}_L t_R \Sigma_t + \bar{\psi}_L b_R \tilde{\Sigma}_b + \text{h.c.}) . \quad (3)$$

where  $\Sigma_t$  and  $\Sigma_b$  are 2 complex doublet scalar fields describing the  $q\bar{q}$  bound states.

$$\Sigma_t = \begin{pmatrix} \Sigma_t^0 \\ \Sigma_t^- \end{pmatrix} \sim t_R \bar{\psi}_L , \quad \tilde{\Sigma}_b = \begin{pmatrix} \Sigma_b^+ \\ -\Sigma_b^{0*} \end{pmatrix} \sim b_R \bar{\psi}_L \quad (4)$$

For the  $\theta$  term, we shall take, in analogy with QCD, the lagrangian form

$$L_\theta = -\frac{\alpha}{4} \left[ i \text{Tr} \left( \ln U - \ln U^\dagger \right) + 2\theta \right]^2 , \quad (5)$$

with

$$U = \begin{pmatrix} \Sigma_t^0 & \Sigma_b^- \\ \Sigma_t^+ & -\Sigma_b^{0*} \end{pmatrix} . \quad (6)$$

This term typically arises as a leading term in a  $1/N$  - expansion.

From Eqs.(2) and (3) it follows that the top and bottom field-dependent masses are given at the tree level by the linear combinations

$$M_t = h_t H^0 + g \Sigma_t^0, \quad M_b = h_b \tilde{H}^0 + g \tilde{\Sigma}_b^0, \quad (7)$$

## 2 Electroweak and isospin symmetry breakings

Without loss of generality, we take the phase of the neutral Higgs field  $H^0$  to be zero. This can always be achieved by performing a suitable electroweak gauge transformation. We write the VEVs of the neutral components of the fields in the form

$$\langle H^0 \rangle = \frac{v}{\sqrt{2}}, \quad \langle \Sigma_t^0 \rangle = \frac{\sigma_t}{\sqrt{2}} e^{i\varphi_t}, \quad \langle \Sigma_b^0 \rangle = \frac{\sigma_b}{\sqrt{2}} e^{i\varphi_b}. \quad (8)$$

Including the radiative corrections (induced by top and bottom quark loops)[9], the effective potential in terms of these VEVs reads

$$V = m_H^2 \frac{v^2}{2} + \frac{m^2}{2} (\sigma_t^2 + \sigma_b^2) - \beta (\mu_t^2 + \mu_b^2) + \lambda (\mu_t^4 + \mu_b^4) + \alpha (\theta - \varphi_t + \varphi_b)^2, \quad (9)$$

where

$$\mu_{t,b}^2 = |\langle M_{t,b} \rangle|^2 = \frac{1}{2} (h_{t,b}^2 v^2 + g^2 \sigma_{t,b}^2 + 2h_{t,b} v g \sigma_{t,b} \cos \varphi_{t,b}), \quad (10)$$

while  $\beta$  and  $\lambda$  are some effective quadratic and quartic couplings. In what follows we shall assume all couplings and parameters in the potential to be real and positive.

Note that the potential which leads to Eq.(9) can be viewed either as an effective renormalizable interaction or as an expansion up to quartic terms in a cut-off theory.

The extrema conditions  $\frac{\partial V}{\partial v} = \frac{\partial V}{\partial \sigma_t} = \frac{\partial V}{\partial \sigma_b} = \frac{\partial V}{\partial \varphi_t} = \frac{\partial V}{\partial \varphi_b} = 0$  imply a system of equations which can be solved in a simple analytical way assuming  $h_t = h_b$ ,  $\alpha \gg \beta m_t^2$  with  $m_t$  the physical mass of the top quark,  $\beta \gg 2\lambda m_t^2$ . In that

case, the presence of a phase  $\theta$  close to  $\frac{\pi}{2}$  induces both isospin breaking and CP violation with <sup>3</sup>

$$\sigma_b \ll \sigma_t \neq 0, \quad v \neq 0, \quad \varphi_t \simeq \sigma_b/\sigma_t, \quad \varphi_b \simeq -\pi/2 + \sigma_b/\sigma_t. \quad (11)$$

### 3 $CP$ violation

Let us now investigate whether this new source of  $CP$  violation can be responsible for what is observed in the  $K^0 - \bar{K}^0$  system. Let us consider the  $3 \times 3$  quark mass matrices

$$M_{u,d} = (h)_{u,d}v + \begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix} g\sigma_{t,b}e^{\pm i\varphi_{t,b}}, \quad (12)$$

with  $(h)_{u,d}$  arbitrary real matrices.

If we neglect  $O(h^2v^2)$  terms,  $(MM^\dagger)_{u,d}$  are diagonalized by the following unitary matrices:

$$U_u \simeq R_u \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{-i\varphi_t} \end{pmatrix}, \quad U_d \simeq R_d \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{i\varphi_b} \end{pmatrix}, \quad (13)$$

with  $R_{u,d}$  orthogonal. In this approximation and using the Cabibbo-Kobayashi-Maskawa parametrization for the quark mixing matrix [5], we get, last but not least,  $\delta_{KM} \simeq -(\varphi_t + \varphi_b) \simeq \pi/2$ . In the nowadays standard parametrization of the Cabibbo-Kobayashi-Maskawa mixing matrix[1], the phenomenology in  $K^0 - \bar{K}^0$  physics requires that the CP-violating phase ( $\delta_{13}$ ) be around  $\frac{\pi}{2}$ . The CP-violating phase in the 2 parametrizations are related by  $\sin \delta_{13} \simeq \frac{\vartheta_2}{\vartheta_{23}} \sin \delta_{KM}$ , in the small mixing angles approximation[13]. From  $|V_{ij}|_{KM} = |V_{ij}|_{standard}$ , the experimental constraint on the ratio

$$\frac{V_{ub}}{V_{cb}} = 0.08 \pm 0.02 \quad (14)$$

require a texture for the  $h$ -matrices such that the mixing angles  $\vartheta_2 > \vartheta_3$ .

---

<sup>3</sup> the question of the eigenvalues of the scalar mass matrix is discussed in Ref.[7]

Therefore, we conclude that our solution leads indeed to a sizeable  $CP$ -violating phase

$$\delta_{13} \simeq \pi/2, \quad (15)$$

which is welcome by phenomenology in  $K^0 - \bar{K}^0$  physics [14].

## References

- [1] *Review of Particle Physics*, R.M. Barnett *et al.*, Phys. Rev. **D 54** (1996) 1.
- [2] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345.
- [3] A. Miransky, M. Tanabashi and K. Yamawaki, Mod. Phys. Lett. **A 4** (1989) 1043; Phys. Lett. **B 221** (1989) 177.
- [4] W.A. Bardeen, C.T. Hill and M. Lindner, Phys. Rev. **D 41** (1990) 1647.
- [5] M. Kobayashi and K. Maskawa, Prog. Theor. Phys. **49** (1973) 652.
- [6] For a recent review, see e.g. R.D. Peccei, *QCD, Strong CP and Axions*, hep-ph/9606475.
- [7] D. Delepine, J.-M. Gerard, R. Gonzalez Felipe, J. Weyers, Phys. Lett. **B411** (1997) 167.
- [8] G. Buchalla, G. Burdman, C.T. Hill and D. Kominis, Phys. Rev. **D 53** (1996) 5185.
- [9] J.P. Fatelo, J.-M. Gérard, T. Hambye and J. Weyers, Phys. Rev. Lett. **74** (1995) 492.
- [10] E. Witten, Ann. Phys. **128** (1980) 363.
- [11] C. Vafa and E. Witten, Nucl. Phys. **B234** (1984) 173.
- [12] D. Delépine, J.-M. Gérard and R. González Felipe, Phys. Lett. **B 372** (1996) 271.
- [13] H. Fritzsch, Phys. Rev. **D 32** (1985) 3058.

- [14] A.J. Buras, M. Jamin and M.E. Lautenbacher, Phys. Lett. **B 389** (1996) 749.